Rectangular Components and Resultant Forces

Force: Is an action that change or tend to change the state of motion of the body upon which it acts. It is a vector quantity that can be represented either mathematically or graphically.

« Resolution of Forces »

1- Rectangular Components:

Let the Force (F) shown in Fig(1) below with the direction (θ)

we can resolve this Force into two components:

1- Horizontal component (F_x) which lies on x-axis.

2- Vertical Component (F_y) which lies on y-axis as shown below.

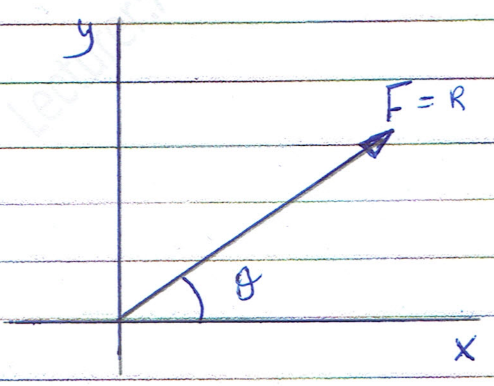


Fig (1)

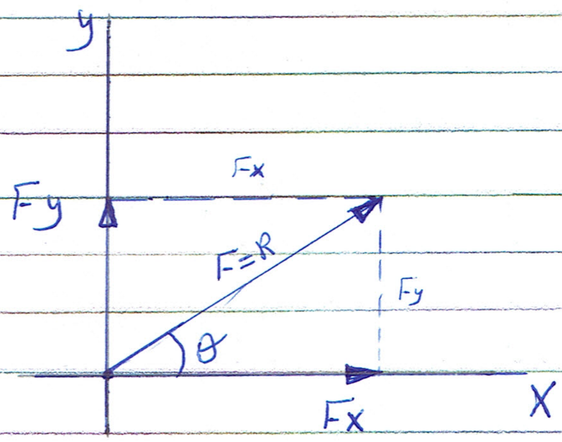
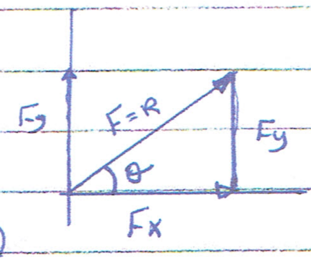


Fig (2)

From Figure beside:



The horizontal component (Fx) determine as:

$$\cos\theta = \frac{F_x \;\text{المجاور}}{F \;\text{الوتر}} \Rightarrow \boxed{F_x = F\cos\theta} \quad \text{---} (1)$$

The vertical Componet determine as:

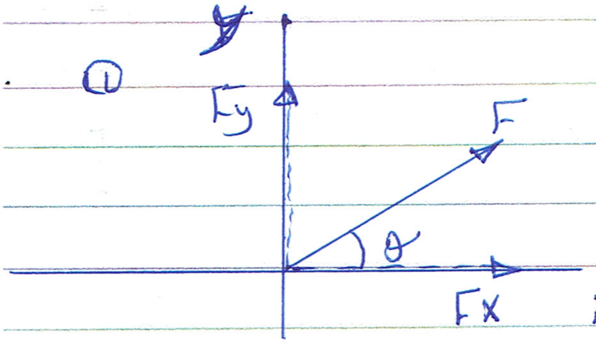
$$\sin\theta = \frac{F_y \;\text{المقابل}}{F \;\text{الوتر}} \Rightarrow \boxed{F_y = F\sin\theta} \quad \text{---} (2)$$

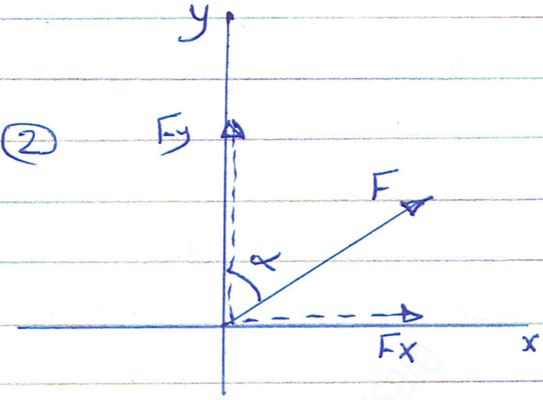
$$\boxed{F = R = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(R_x)^2 + (R_y)^2}} \quad \text{---} (3)$$

The direction of Force (F) is

$$\boxed{\tan\theta = \frac{F_y \;\text{مقابل}}{F_x \;\text{مجاور}} \Rightarrow \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)} \quad \text{---} (4)$$

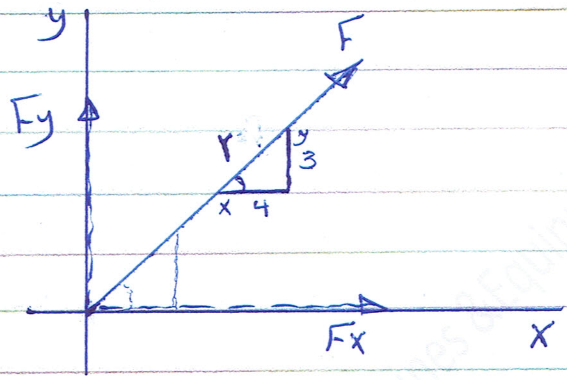
Rectangular components

① 

② 

$$F_x = F \cos\theta \qquad\qquad F_x = F \sin\alpha$$

$$F_y = F \sin\theta \qquad\qquad F_y = F \cos\alpha$$



$$r = \sqrt{(x)^2 + (y)^2} \Rightarrow = \sqrt{(4)^2 + (3)^2} = 5\ m$$

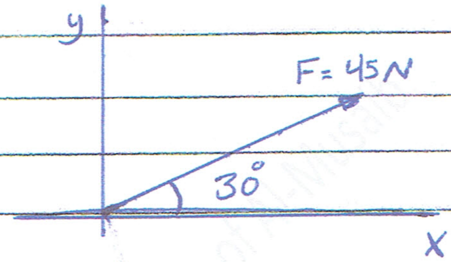
$$F_x = F \cos\theta \qquad \left(\cos\theta = \frac{4 \text{ مجاور}}{5 \text{ وتر}}\right)$$

$$\therefore F_x = F \times \frac{4}{5}$$

$$F_y = F \times \sin\theta \qquad \left(\sin\theta = \frac{3 \text{ مقابل}}{5 \text{ وتر}}\right)$$

$$\therefore F_y = F \times \frac{3}{5}$$

Example 1: Resolve the force shown in fig below into two components, one along x-axis and the other along y-axis.



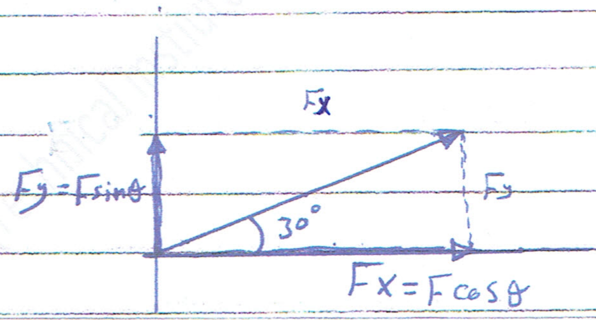
Solution:

① $F_x = F\cos\theta$

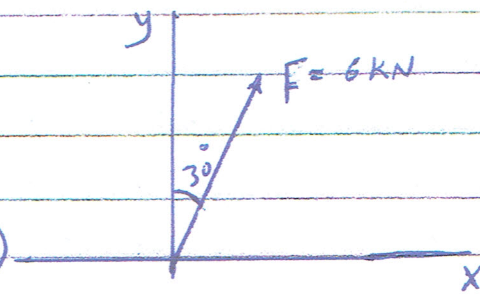
$= 45\cos 30 = 38.98\ N$

② $F_y = F\sin\theta$

$45\sin 30 = 22.5\ N$



Example 2: For the Figure below, Determine the Components of a force (F) along X and y axes



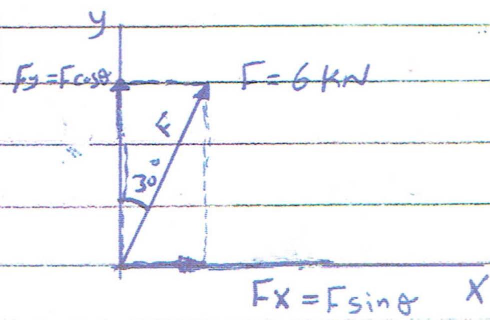
Solution:

$F_x = F\sin\theta \quad \left(\sin\theta = \dfrac{F_x}{F} \quad \dfrac{\text{مقابل}}{\text{وتر}}\right)$

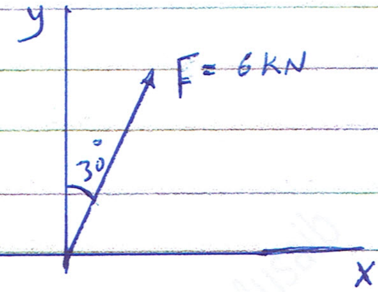
$= 6\sin 30 = 3\ KN$

$F_y = F\cos\theta \quad \left(\cos\theta = \dfrac{F_y}{F} \quad \dfrac{\text{مجاور}}{\text{وتر}}\right)$

$= 6\cos 30 = 5.2\ KN$



Example 2: For the Figure below, Determine the Components of a force (F) along X and y axes

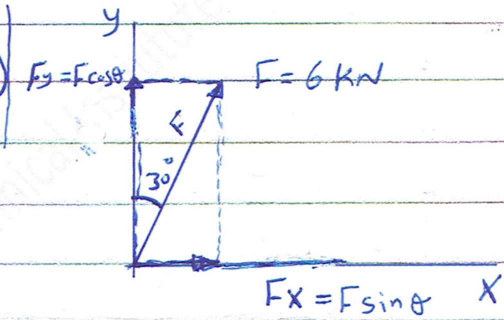


Solution:

$F_x = F\sin\theta \qquad \left(\sin\theta = \frac{F_x}{F} \; \frac{\text{مقابل}}{\text{وتر}}\right)$

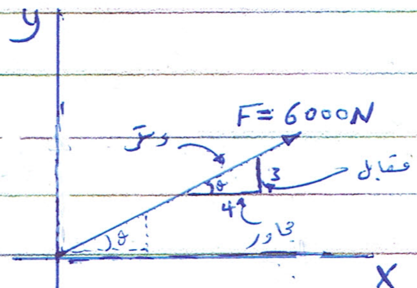
$= 6\sin 30 = 3\ KN$

$F_y = F\cos\theta \qquad \left(\cos\theta = \frac{F_y}{F} \; \frac{\text{مجاور}}{\text{وتر}}\right)$



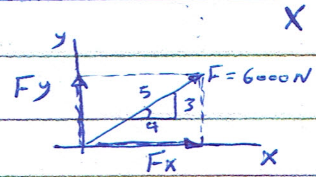
$= 6\cos 30 = 5.2\ KN$

Example 3: Determine the horizontal and vertical Components (Fx, Fy) of force (F) shown in Figure below.



Solution:

$r = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9} = 5$



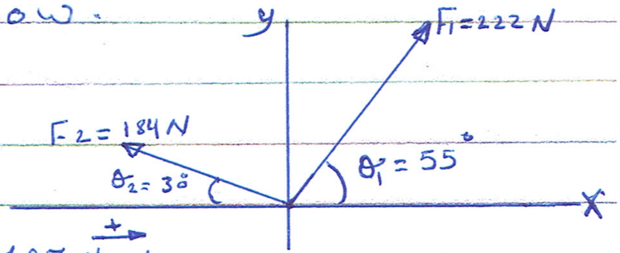
① The horizontal Component is:-

$F_x = F\cos\theta \Rightarrow F_x = 6000 \times \frac{4}{5} = 4800N \qquad \left(\cos\theta = \frac{4}{5} \; \frac{\text{مجاور}}{\text{وتر}}\right)$

② The vertical Component is:-

$F_y = F\sin\theta \Rightarrow F_y = 6000 \times \frac{3}{5} = 3600\,N \qquad \left(\sin\theta = \frac{3}{5} \; \frac{\text{مقابل}}{\text{وتر}}\right)$

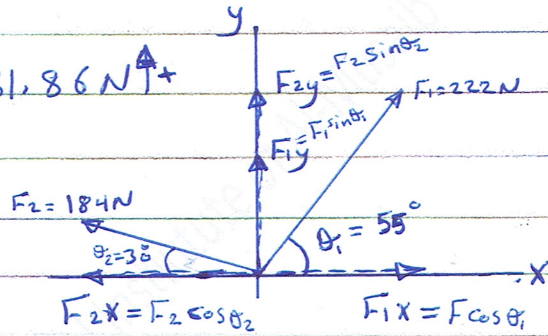
Example 4: Find the Components of forces (F_1, F_2) shown in figure below.



<u>Solution:</u>

$F_1x = F_1 \cos\theta_1 \Rightarrow = 222 \cos 55^\circ = 127.4\ N \xrightarrow{+}$

$F_1y = F_1 \sin\theta_1 \Rightarrow = 222 \sin 55^\circ = 181.86\ N \uparrow +$

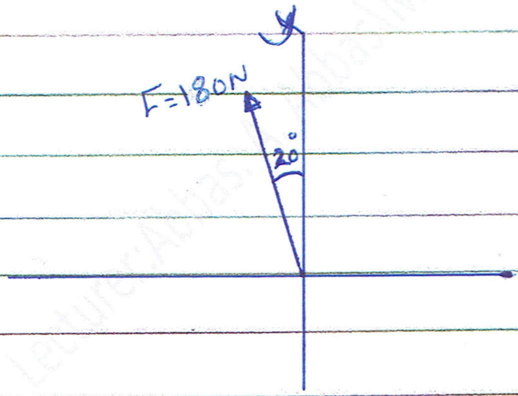


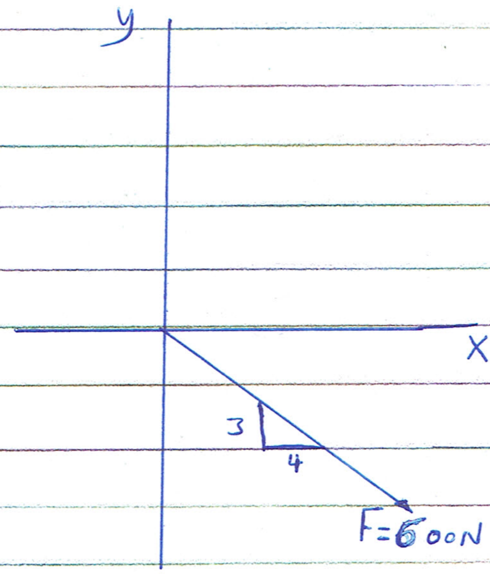
$F_2x = F_2 \cos\theta_2 = 184 \cos 30^\circ = 159.35\ N \leftarrow$

$F_2y = F_2 \sin\theta_2 = 184 \sin 30 = 92\ N \uparrow +$

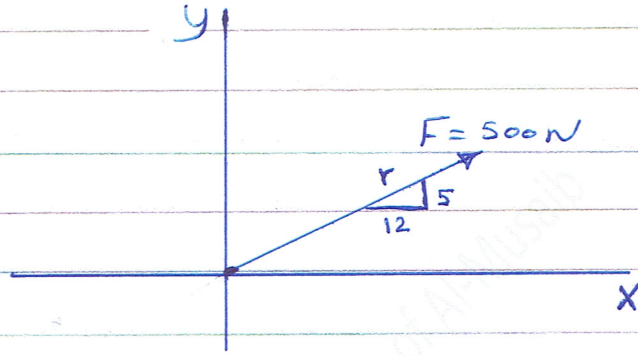
<u>Homework</u>

H.W: For the Figures below, Determine the components of a Forces (F) along X and y axes (F_x, F_y)?





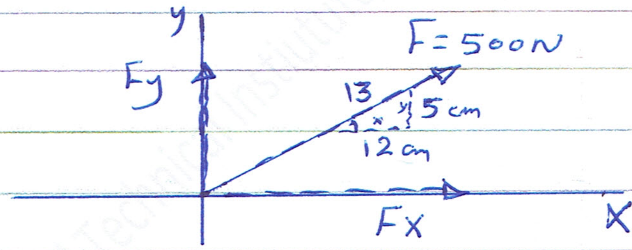
Example 5: Resolve the 500 N Force shown in Fig below into two Components along (X and y) direction.



Solution:

$$r = \sqrt{(x)^2 + (y)^2}$$

$$= \sqrt{(12)^2 + (5)^2} = 13 \text{ cm}$$



$$F_x = F \cos\theta$$

مجاور

$$\therefore F_x = 500 * \frac{12}{13} = 461.6 \xrightarrow{+}$$

وتر

$$F_y = F \sin\theta$$

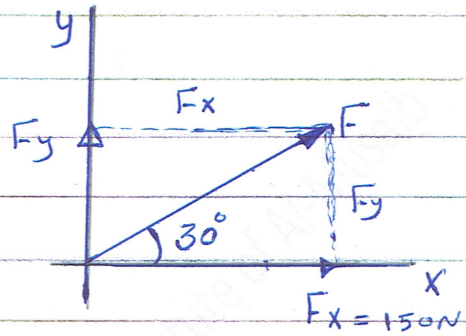
مقابل

$$\therefore F_y = 500 * \frac{5}{13} = 192.3 \text{ N} \uparrow +$$

وتر

Example 6: If the horizontal component of force equal to 150N as shown in figure below, Find

① The Force F ② Vertical Component (F_y)



Solution:

$$\cos\theta = \frac{F_x}{F} \qquad \frac{\text{مجاور}}{\text{وتر}}$$

$$\therefore F\cos\theta = F_x$$

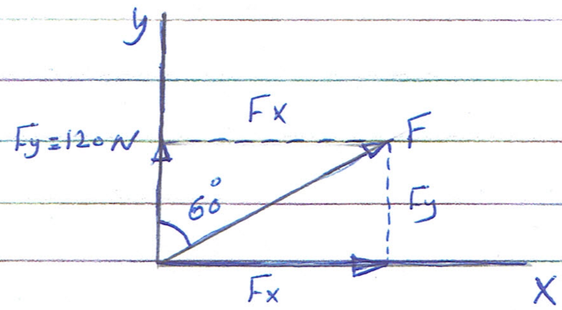
①

$$\therefore F = \frac{F_x}{\cos\theta} \Rightarrow = \frac{150}{\cos 30} = 173.2\ N$$

② $$\sin\theta = \frac{F_y}{F} \qquad \frac{\text{مقابل}}{\text{وتر}}$$

$$\therefore F_y = F\sin\theta \Rightarrow = 173.2 \sin 30 = 86.6\ N$$

Example 7: If the vertical component of force 120N as shown below, find: ① The Force F
② horizontal Component



Sol:

$$\cos\theta = \frac{F_y}{F} \qquad \frac{\text{مجاور}}{\text{وتر}}$$

$$F_y = F\cos\theta$$

①

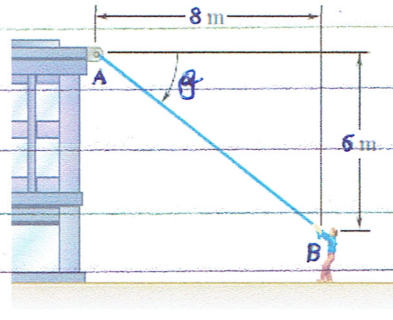
$$\therefore F = \frac{F_y}{\cos\theta} = \frac{120}{\cos 60} = 240\ N$$

②

$$\sin\theta = \frac{F_x}{F} \Rightarrow F_x = F\sin\theta$$

$$\therefore F_x = 240 \sin 60 = 207.85\ N$$

Example 8: A man pulls with a force of 300 N on a rope attached to a building, as shown in Fig. below. What are the horizontal and vertical components of the force exerted by the rope at point *A*? and Find the angle θ.



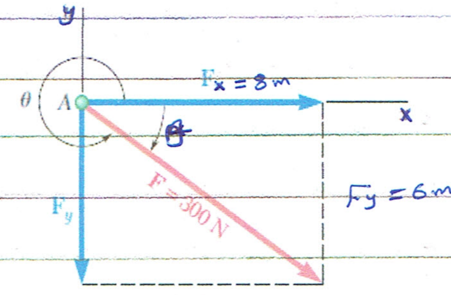
(a)

Solution:

$F_x = F\cos\theta \qquad \left(\cos\theta = \frac{F_x \; \text{مجاور}}{F \; \text{وتر}}\right)$

$F_y = F\sin\theta \qquad \left(\sin\theta = \frac{F_y \; \text{مقابل}}{F \; \text{وتر}}\right)$

$\overrightarrow{AB} = \sqrt{(F_x)^2 + (F_y)^2} = \sqrt{(8)^2 + (6)^2} = 10\,m$



(b)

① $\therefore F_x = 300 * \left(\frac{8 \; \text{مجاور}}{10 \; \text{وتر}}\right)_{\cos\theta} = \boxed{240\,N} \xrightarrow{+}$

② $\therefore F_y = 300 * \left(\frac{6 \; \text{مقابل}}{10 \; \text{وتر}}\right)_{\sin\theta} = \boxed{180\,N} \downarrow -$

Either

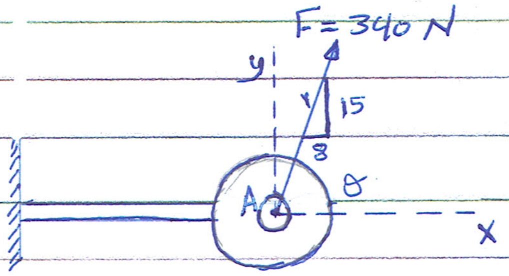
③ $\tan\theta = \frac{F_y \; \text{مقابل}}{F_x \; \text{مجاور}} \Rightarrow \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

$= \tan^{-1}\left(\frac{180}{240}\right) = \boxed{36.87^\circ}$

or

$\tan\theta = \frac{6}{8} \Rightarrow \theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$

Question: Determine the horizontal and vertical components of the force shown below, and find the value of the angle θ.

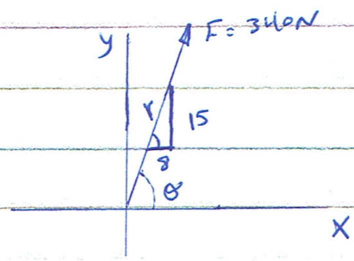


Answer:

$F_x = 160\,N \xrightarrow{+}$

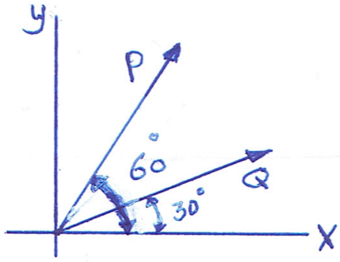
$F_y = 300\,N \uparrow +$

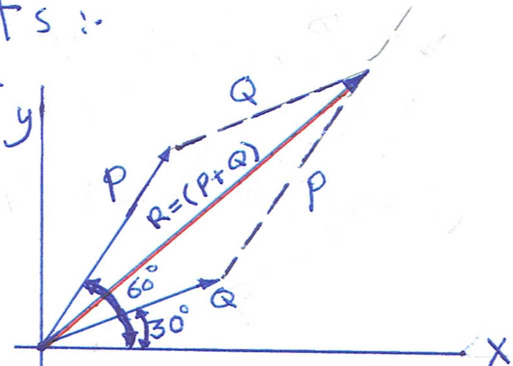
$\theta = 62^\circ$



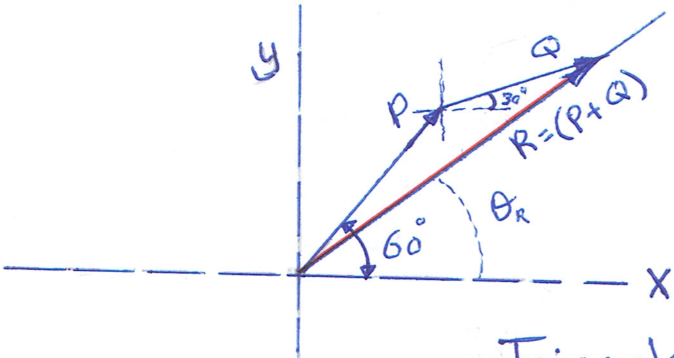
② Non-Rectangular Components :-

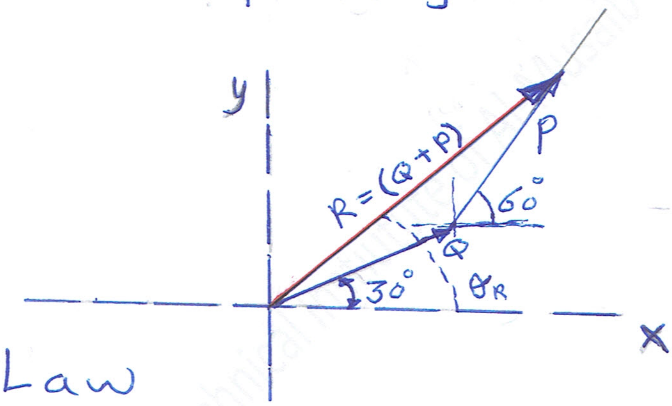
The sum of two vectors- (Two Forces)





Parallelogram Law



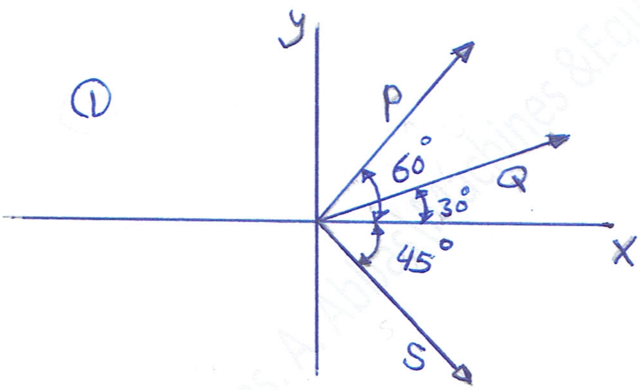


Triangle Law

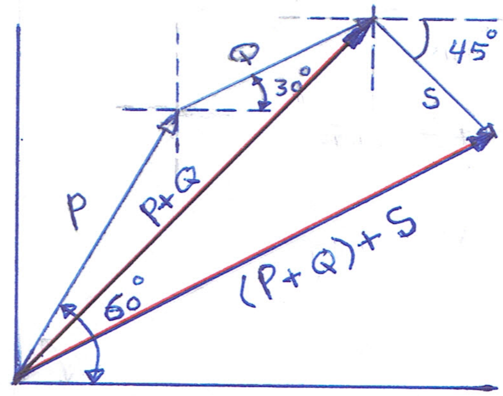
The sum of three or more vectors:-
(more than Two Forces)

$$P+Q+S=(P+Q)+S=P+(Q+S)$$

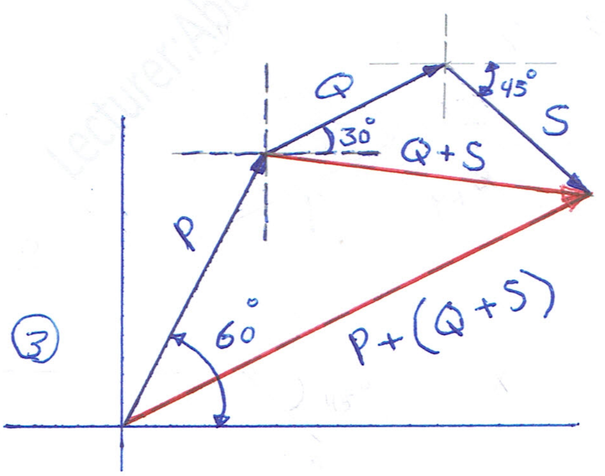
①



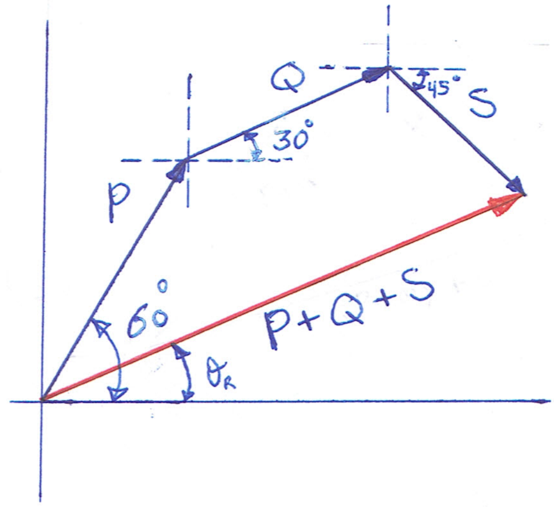
②



③

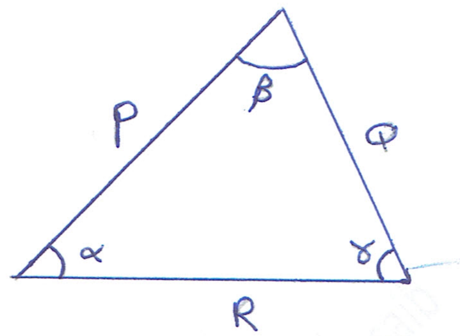


④



Law of Sine

$$\frac{P}{\sin \gamma} = \frac{Q}{\sin \alpha} = \frac{R}{\sin \beta}$$

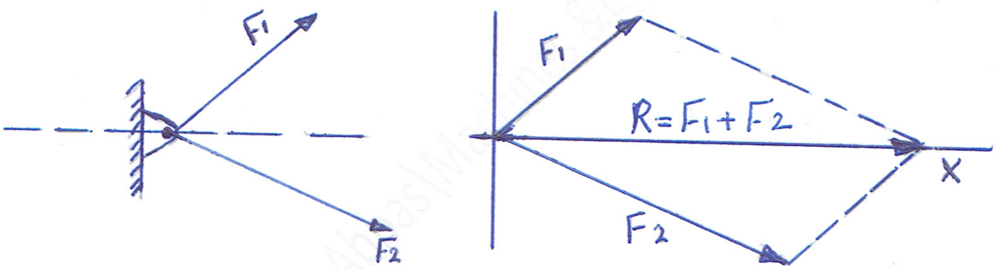


Law of cosine

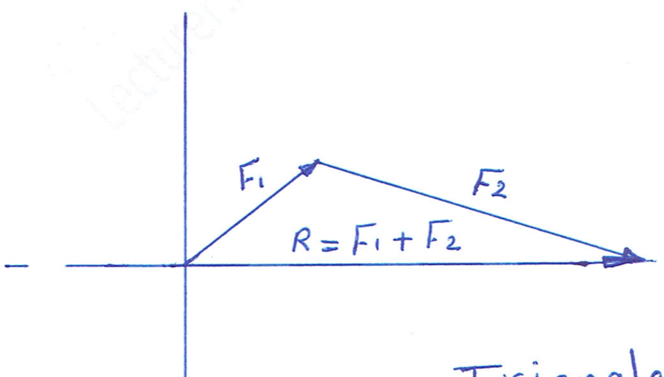
$$R = \sqrt{(P)^2 + (Q)^2 - 2PQ\cos\beta}$$

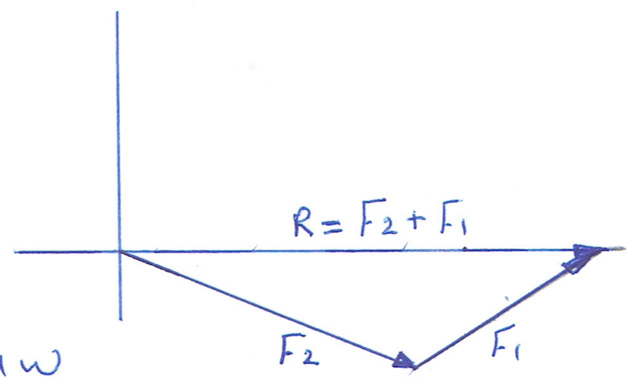
$$P = \sqrt{(Q)^2 + (R)^2 - 2QR\cos\gamma}$$

$$Q = \sqrt{(P)^2 + (R)^2 - 2PR\cos\alpha}$$

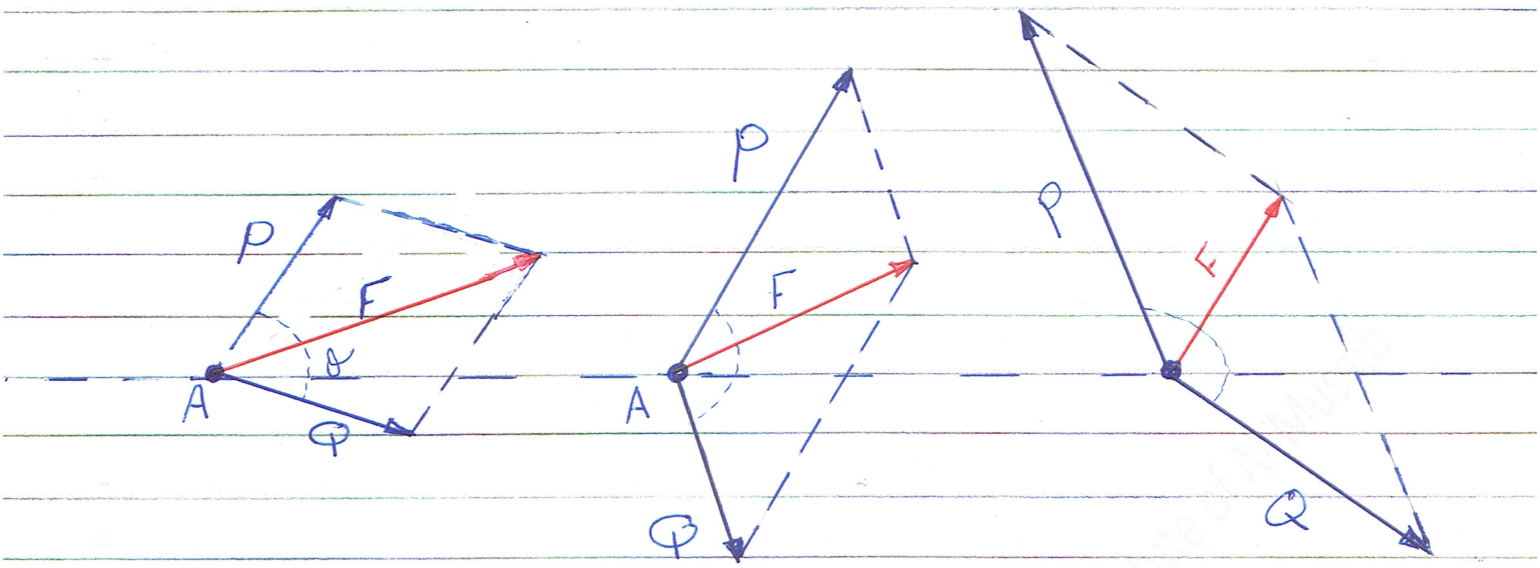


Parallelogram Law

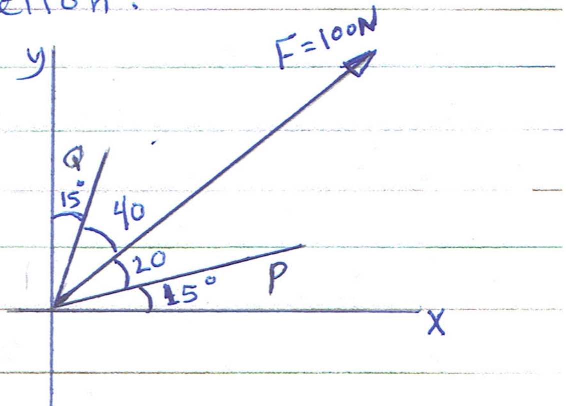




Triangle Law



Example 9: Resolve the 100N Force shown in the Figure below into two components along P and Q direction.



(a)

Solution:

$$\frac{P}{\sin\gamma} = \frac{Q}{\sin\alpha} = \frac{F}{\sin\beta}$$

From the Fig (b) below we find that :-

$\beta = 120°$ $\alpha = 20$

$\gamma = \boxed{180} - (120 + 20) = 40°$

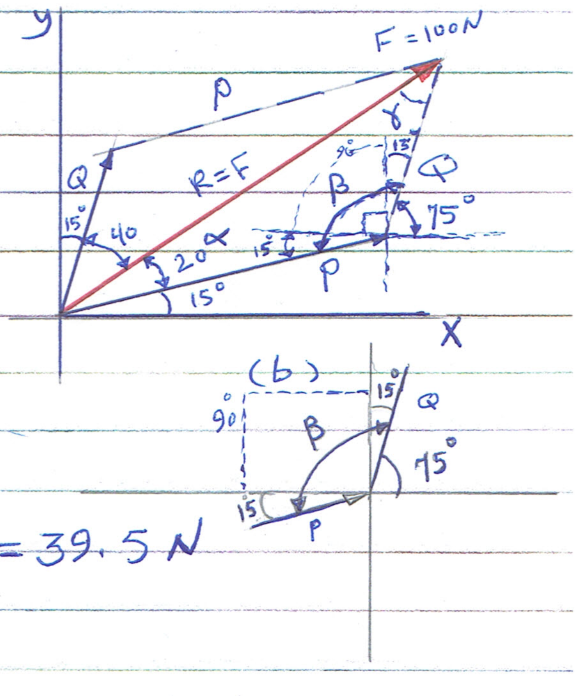
∠ sum of triangle angles (constant)

$$\frac{P}{\sin\gamma} = \frac{F}{\sin\beta}$$

$$\frac{P}{\sin 40} = \frac{100}{\sin 120} \Rightarrow P = \frac{100 \sin 40}{\sin 120} = 74.23\,N$$

$$\frac{Q}{\sin} = \frac{F}{\sin\beta}$$

$$\frac{Q}{\sin 20} = \frac{100}{\sin 120} \Rightarrow Q = \frac{100 \sin 20}{\sin 120} = 39.5\,N$$



(b)

« Resultant of Forces System »

Resultant :- It's represent an equivalent Force, that equal to the effects of all the external Forces acted on the body

* we use a letter "R" to represent the resultant Force.

* A resultant force (R) is a vector quantity, so we need to know both, the magnitude and direction of the resultant force.

We will find out the resultant force for many Forces acting on a rigid body by using the following equations:

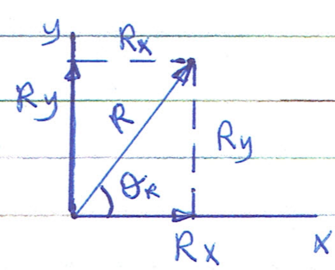
$$R_x = \sum F_x = F_{1x} \mp F_{2x} \mp F_{3x} \mp F_{nx} ---$$

$$R_y = \sum F_y = F_{1y} \mp F_{2y} \mp F_{3y} \mp F_{ny} ---$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = \sqrt{(R_x)^2 + (R_y)^2}$$

The direction of resultant Force determined as:-

$$\tan\theta = \frac{R_y \ \text{(المقابل)}}{R_x \ \text{(المجاور)}} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y \ \text{(المقابل)}}{R_x \ \text{(المجاور)}}\right)$$



We can use the following methods to determine the resultant of forces

A- Graphical method

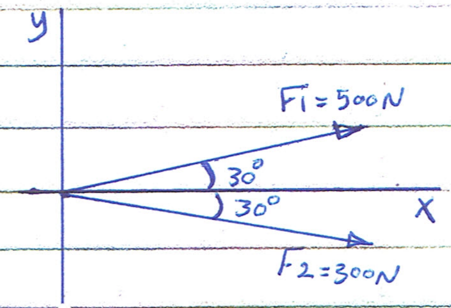
B- Analytical method

C- Trigonometric method

① Graphical method

لأيجاد محصلة القوى بالطريقة البيانية نتبع الخطوات التالية :

Example 10: Determine the resultan of the forces system shown below by using graphical method.



solution:

From the Figure below we find:

$R = 7\ cm \Rightarrow R = 7 * 100 = \boxed{700\ N}$

وبعد قياس الزاوية θ_R نجد ان الزاوية تقريباً 8.3° وتمثل اتجاه المحصلة

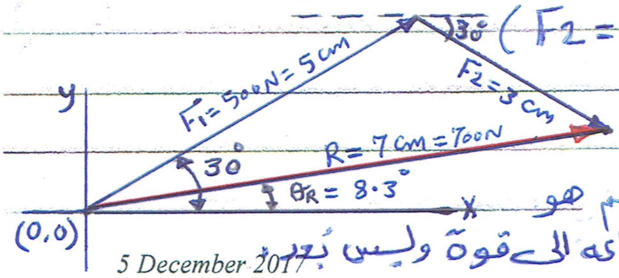
① نختار مقياس رسم مناسب كأن يكون لهذا السؤال ← $100\ kN = 1\ cm$

$\therefore F_1 = \frac{500\ kN}{100} = 5\ cm$

$\therefore F_2 = \frac{300\ kN}{100} = 3\ cm$

② نبدأ الرسم للقوة F_1 من نقطة ال $(0,0)$ الواقعة على محوري (X, Y) وبالطول المقابل لهذه القوة $(F_1 = 5\ cm)$ وبنفس زاوية الميل بأتجاه خط الأفق (محور X)

③ من نقطة نهاية خط القوة F_1 نبدأ برسم القوة F_2 بنفس زاوية الميل لهذه القوة وبالطول المقابل لها $(F_2 = 3cm)$ وبأتجاه خط الافق (محور X)



④ نقوم برسم خط مستقيم بين نقطة بداية القوة F_1 ونقطة نهاية القوة F_2 فيكون طول ذلك المستقيم هو محصلة لنظام القوى (F_1, F_2) بعد ضربه بمقياس الرسم لأرجاعه الى قوة وليس بُعد

محصلة لعدة قوى مستوية متلاقية في نقطة واحدة ~~بيانياً~~ وتحليلياً .

• جمع قوتين تحليلياً :- ② Analytical method

أ- نحلل كل قوة الى مركبتين (افقية وعمودية)

ب- نجمع المركبات التي تقع على محور (X) ونحصل على مركبة المحصلة باتجاه محور (X) والتي هي $(\Sigma F_x = R_x)$.

ج- نجمع المركبات التي تقع على محور (y) ونحصل على مركبة المحصلة باتجاه (y) والتي هي $(\Sigma F_y = R_y)$.

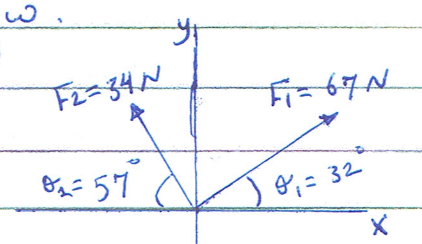
د- نحصل على المحصلة من خلال المعادلة التالية:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(R_x)^2 + (R_y)^2}$$

هـ- نحصل على اتجاه المحصلة (زاوية ميلها) من خلال المعادلة التالية:

$$\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Example 11: Find the resultant ① and its direction ② of two Forces shown below.
(by using analytical method)



Solution:

$R_x = \Sigma F_x = F_{1x} + F_{2x}$

قوة سالبة

$= F_1 \cos\theta_1 - F_2 \cos\theta_2$

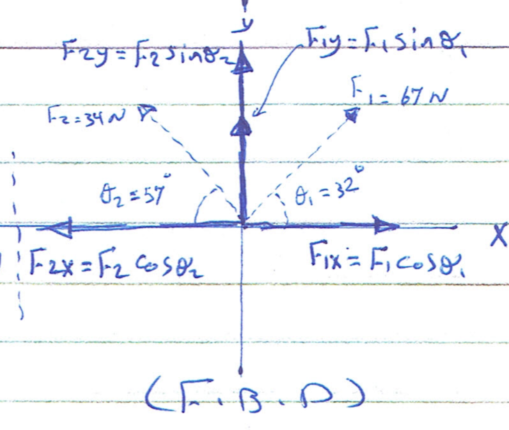
56.82 18.52

$= 67\cos 32 - 34\cos 57 = 38.3\,N$

$R_y = \Sigma F_y = F_{1y} + F_{2y}$

$= F_1 \sin\theta_1 + F_2 \sin\theta_2$

$= 67\sin 32 + 34\sin 57 = 64\,N$

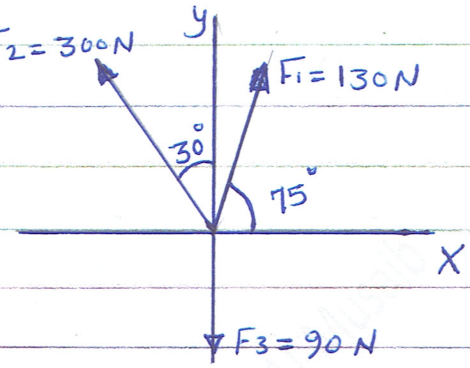


① $\therefore R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(38.3)^2 + (64)^2} = 74.59\,N$

(F.B.D)

② $\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}\left(\frac{64}{38.3}\right) = 59.1°$

Example (12): Find the resultant (R) and its direction of the Forces system shown below. (by using analytical method).



Solution:-

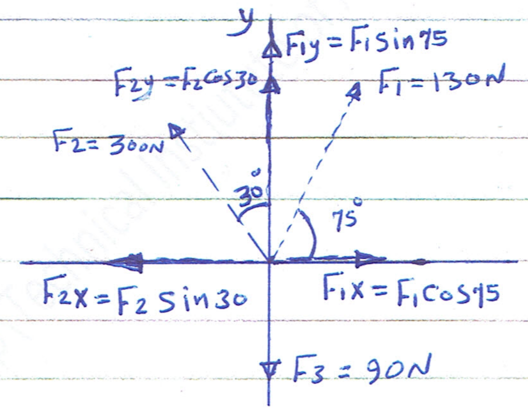
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$R_x = \sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= F_1 \cos 75 - F_2 \sin 30 + 0$$

$$= 130 \cos 75 - 300 \sin 30$$

$$= 33.65 - 150 = \boxed{-116.35N}$$



$$R_y = \sum F_y = F_{1y} + F_{2y} + F_{3y}$$

$$= F_1 \sin 75 + F_2 \cos 30 -$$

$$= 130 \sin 75 + 300 \cos 30 - 90$$

$$= 125.58 + 259.8 - 90 = \boxed{295.38N}$$

$$\therefore R = \sqrt{(-116.35)^2 + (295.38)^2} = 317.5N$$

The direction of R is:-

$$\tan\theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{295.38}{-116.35}\right) = |-68.5| = 68.5^\circ$$

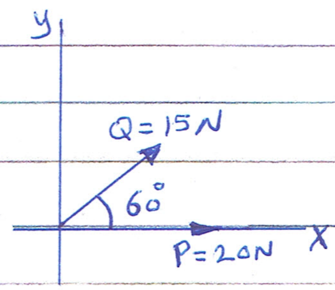
Example 13: Find the resultant (R) of the Forces shown below (by using trigonometrical method)

Solution:

From the Figure (b) below:

$\beta = 120^\circ$

From cosine Law

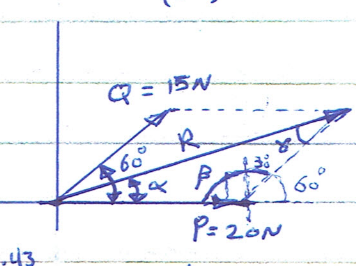


(a)

$$\therefore R = \sqrt{(P)^2 + (Q)^2 - 2PQ\cos\beta}$$

$$= \sqrt{(20)^2 + (15)^2 - 2 * 20 * 15 \cos 120}$$

$$= \boxed{30.5N}$$



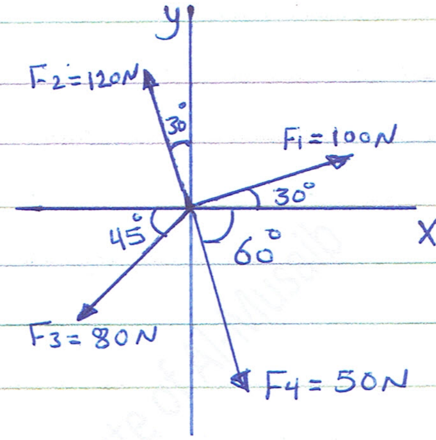
(b)

From sine Law

The direction is $\alpha \Rightarrow \frac{R}{\sin\beta} = \frac{Q}{\sin\alpha} \Rightarrow \frac{30.5}{\sin 120} = \frac{15}{\sin\alpha} \Rightarrow \sin\alpha = \frac{15 \sin 120}{30.5} = 0.43$

$\therefore \alpha = \sin^{-1}(0.43) = 25.5^\circ$

Example 14: For the Figure below, Find the resultant and its direction for the Forces system.



Solution:

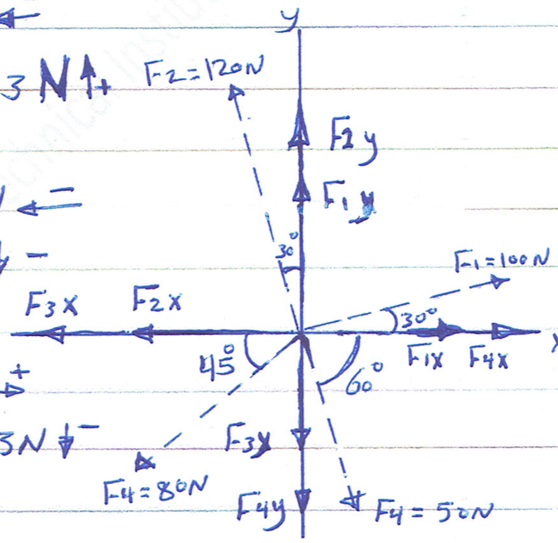
$$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2} = \sqrt{(Rx)^2 + (Ry)^2}$$

$F_1x = F_1 \cos\theta_1 \Rightarrow = 100\cos 30 = 86.6N \xrightarrow{+}$

$F_1y = F_1 \sin\theta_1 \Rightarrow = 100\sin 30 = 50N \uparrow +$

$F_2x = F_2 \sin\theta_2 \Rightarrow = 120\sin 30 = 60N \xleftarrow{-}$

$F_2y = F_2 \cos\theta_2 \Rightarrow = 120\cos 30 = 103.93N \uparrow +$



$F_3x = F_3 \cos\theta_3 \Rightarrow = 80\cos 45 = 56.6N \xleftarrow{-}$

$F_3y = F_3 \sin\theta_3 \Rightarrow = 80\sin 45 = 56.6N \downarrow -$

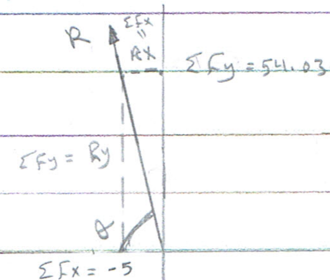
$F_4x = F_4 \cos\theta_4 \Rightarrow = 50\cos 60 = 25N \xrightarrow{+}$

$F_4y = F_4 \sin\theta_4 \Rightarrow = 50\sin 60 = 43.3N \downarrow -$

$$Rx = \Sigma Fx = F_1x + F_2x + F_3x + F_4x$$
$$= 86.6 - 60 - 56.6 + 25 = \boxed{-5\ N}$$

$$Ry = \Sigma Fy = F_1y + F_2y + F_3y + F_4y$$
$$= 50 + 103.93 - 56.6 - 43.3 = \boxed{54.03\ N}$$

$$\therefore R = \sqrt{(-5)^2 + (54.03)^2} = \boxed{54.3\ N}$$



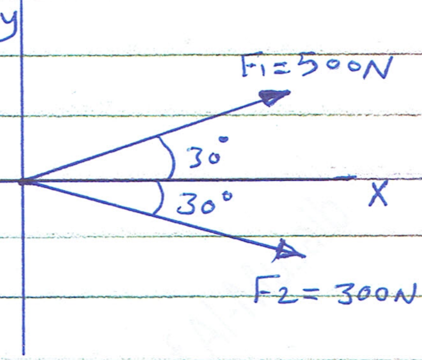
The direction of the Resultant (R) is:

$$\tan\theta = \left|\frac{Fy}{Fx}\right| \Rightarrow \theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$$

مقابل
مجاور

$$= \tan^{-1}\left|\left(\frac{54.03}{-5}\right)\right| = 84.8 \simeq \boxed{85^\circ}$$

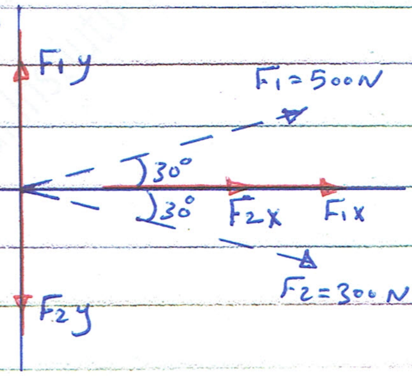
Example 15: Resolve the former question by using analytical method to find the resultant (R) and its direction.



Solution:

$$R = \sqrt{(Rx)^2 + (Ry)^2}$$

$$Rx = \sum Fx = F_{1X} + F_{2X}$$
$$= 500\cos 30 + 300\cos 30$$
$$= 433 + 260 = \boxed{693\,N}$$



$$Ry = \sum Fy = F_{1y} + F_{2y}$$
$$= 500\sin 30 - 300\sin 30$$
$$= 250 - 150 = \boxed{100N}$$

$$\therefore R = \sqrt{(693)^2 + (100)^2} = \boxed{700\,N}$$

The direction is : $\theta R = \tan^{-1}\left(\frac{Ry}{Rx}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{100}{693}\right) = \boxed{8.3^\circ}$

② Graphical method

50 KN = 1 cm

100 KN = 1 Cm ← لهذا السؤال ① نختار مقياس رسم مناسب كأن يكون

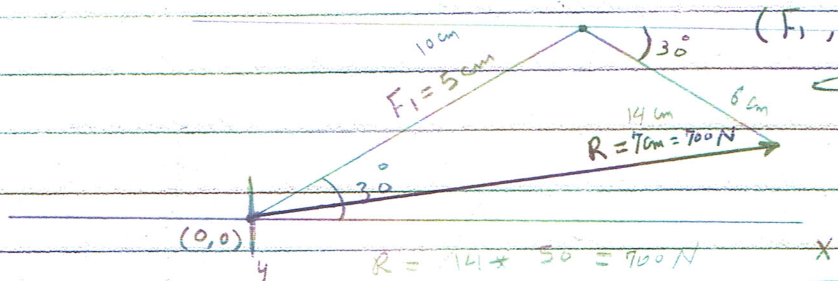
$\therefore F_1 = \frac{500kN}{100\ \ 50} = 5\ \ 10\,cm$

$F_2 = \frac{300kN}{100\ \ 50} = 3\,cm\ \ 6\,cm$

② نبدأ الرسم للقوة F_1 من نقطة الـ (0,0) على محوري (x و y) وبالطول المقابل لهذه القوة ($F_1 = 5\,cm$) وبنفس زاوية الميل ((بأعتماد خط الافق (محور x))

③ من نقطة نهاية خط القوة F_1 نبدأ برسم القوة F_2 وبنفس زاوية الميل لهذه القوة، وبالطول المقابل لها ($F_2 = 3\,cm$) ((بأعتماد خط الافق (محور x))

④ نقوم برسم خط مستقيم بين نقطة بداية القوة F_1، ونقطة نهاية القوة F_2 ويكون طول ذلك الخط هو R محصلة لنظام القوى (F_1, F_2) بعد ضربه بمقياس الرسم لإرجاعه الى قوة وليس بعد.

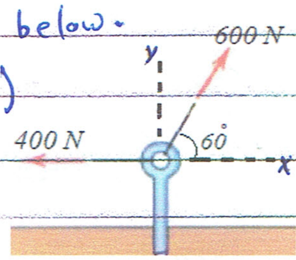


R = 14 * 50 = 700 N

$\therefore R = 7\,cm * 100 = \boxed{700\,N}$ same answer of the analytical method That shown above

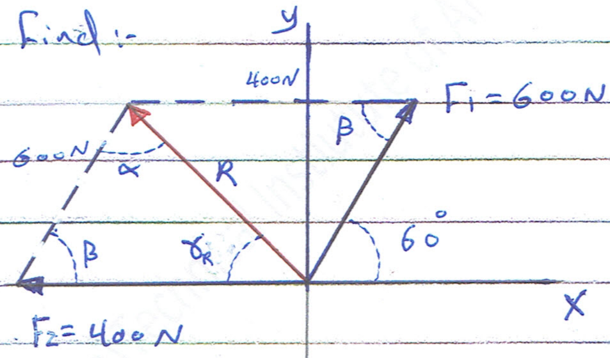
③ Trigonometric method

Example : Determine the resultant (R) and its direction of the forces system shown below. by using (Trigonometrical method)



solution:

From the Fig beside we find:-



$\beta = 60^\circ$ (Alternately)

From Cosine Law

$$R = \sqrt{(F_1)^2 + (F_2)^2 - 2 * F_1 * F_2 \cos\beta}$$

$$= \sqrt{(600)^2 + (400)^2 - 2 * 600 * (400) \cos 60}$$

$\therefore R = 529.2 \ N$

From Sine Law

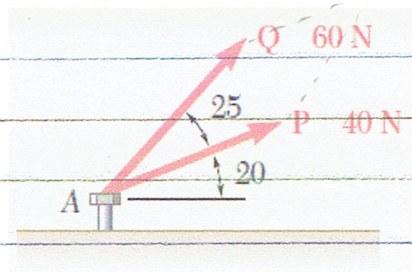
$$\boxed{\frac{R}{\sin\beta} = \frac{F_1}{\sin\gamma}} = \frac{F_2}{\sin\alpha}$$

$$\frac{R}{\sin\beta} = \frac{F_1}{\sin\gamma} \Rightarrow \frac{529.2}{\sin 60} = \frac{600}{\sin\gamma}$$

$$\sin\gamma = \frac{600 \sin 60}{529.2} = 0.981$$

$\therefore \gamma = \sin^{-1}(0.981) = 78.9 \simeq 79^\circ$ The direction of (R)

Example[16]: The two forces P and Q act on a bolt as shown below. Determine their resultant and its direction, by using:-



① Trigonometrical method.
② Analytical method.

Solution:-

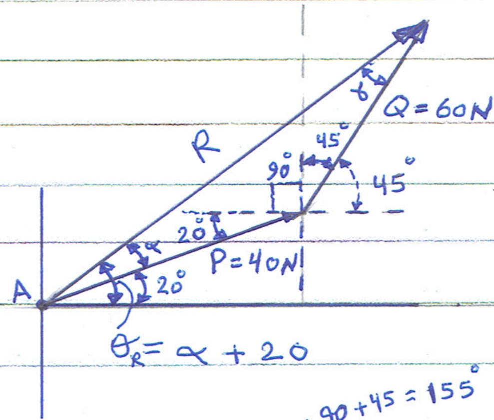
① Trigonometrical Solution:

From the Cos Law

$$R = \sqrt{(P)^2 + (Q)^2 - 2PQ\cos\beta}$$

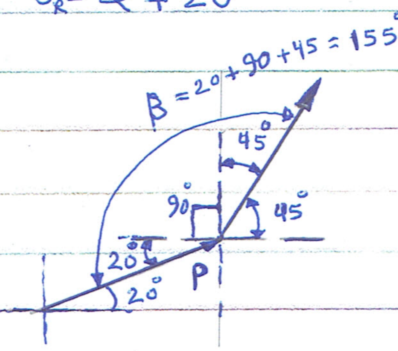
From the Fig beside

$\beta = 20 + 90 + 45 = 155^\circ$



$$\therefore R = \sqrt{(40)^2 + (60)^2 - 2 * 40 * 60 * \cos\beta}$$

$$= \boxed{97.8\ N}$$



From the sin Law

$$\frac{P}{\sin\gamma} = \frac{Q}{\sin\alpha} = \frac{R}{\sin\beta}$$

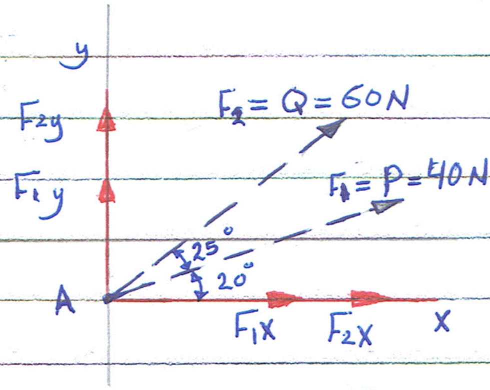
$$Q\sin\beta = R\sin\alpha$$

$$\sin\alpha = \frac{Q\sin\beta}{R} \Rightarrow \sin\alpha = \frac{60\sin 155}{97.8} = 0.26$$

$$\therefore \alpha = \sin^{-1}(0.26) = 15^\circ$$

$$\therefore \theta_R = \overset{\alpha}{15} \overset{+}{+} 20 = 35^\circ$$ The direction of the Resultant (R)

② Analytical solution



$$R = \sqrt{(R_x)^2 + (R_y)}$$

$$R_x = \Sigma F_x = F_{1x} + F_{2x}$$

$$R_y = \Sigma F_y = F_{1y} + F_{2y}$$

$$F_{1x} = F_1 \cos\theta_1 \Rightarrow = 40 \cos 20° = \boxed{37.6\ N} \xrightarrow{+}$$

$$F_{1y} = F_1 \sin\theta_1 \Rightarrow = 40 \sin 20 = \boxed{13.7\ N} \uparrow +$$

$$F_{2x} = F_2 \cos\theta_2 \Rightarrow = 60 \cos 45 = \boxed{42.5\ N} \xrightarrow{+}$$

$$F_{2y} = F_2 \sin\theta_2 \Rightarrow = 60 \sin 45 = \boxed{42.5\ N} \uparrow +$$

$$\therefore R_x = \Sigma F_x = 37.6 + 42.5 = \boxed{80.1\ N}$$

$$\therefore R_y = \Sigma F_y = 13.7 + 42.5 = \boxed{56.2\ N}$$

① $$\therefore R = \sqrt{(80.1) + (56.2)^2} = \boxed{97.8\ N}$$

The direction of (R) is:

② $$\tan\theta = \frac{R_y}{R_y}$$

$$\therefore \theta_R = \tan^{-1}\left(\frac{56.2}{80.1}\right) = \boxed{35°}$$

The direction of R