((Equilibrium))

Equilibrium: Is that condition which has a resultant equals zero, so the body will remain in rest or move with constant velocity.

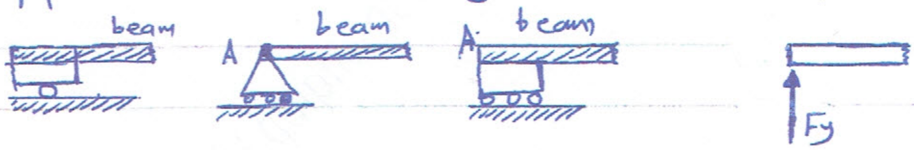
Conditions of Equilibrium:-

1- Resutant Force = zero ⇒ $\sum F_x = 0$, $\sum F_y = 0$

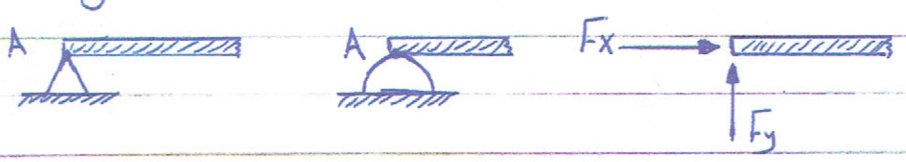
2- sum of moment = zero ⇒ $\sum M_o = 0$

Types of supports:-

① Roller support — يعوض عنها بقوة واحدة عمودية على سطح الاسناد

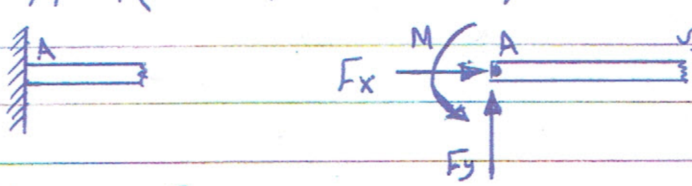


② Pin suppor (Hinge support) — يعوض عنه بقوتين افقية وعمودية بغض النظر عن الجزء المسند

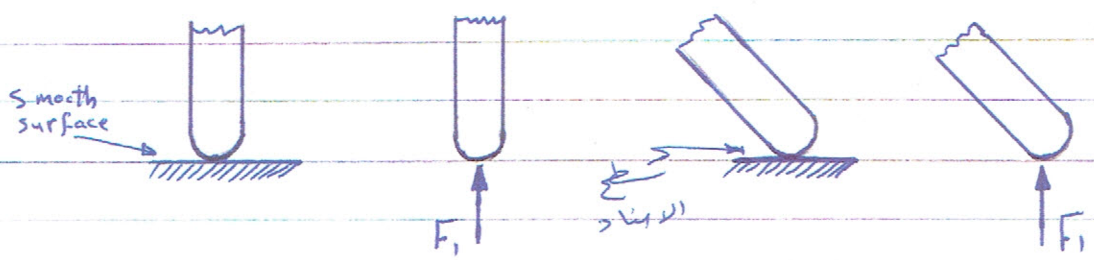


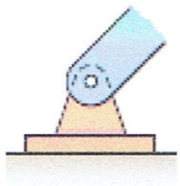
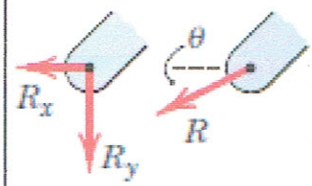
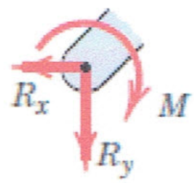
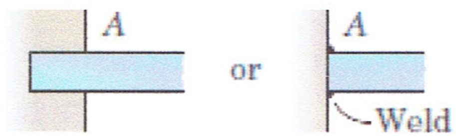
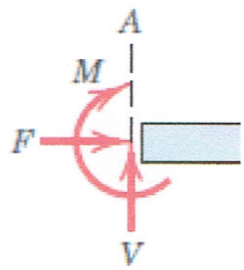
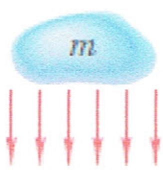
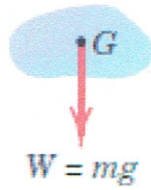
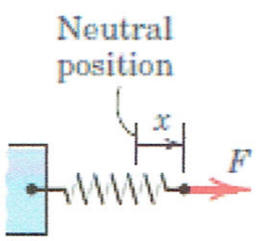
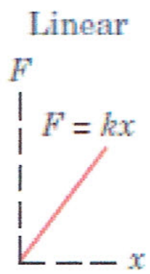
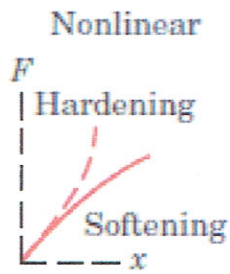
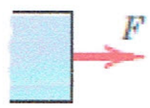
③ Fixed support (Cantilever beam) — يعوض عنه بثلاث ردود افعال

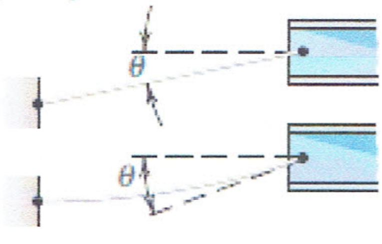
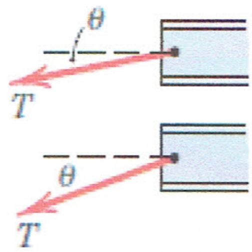
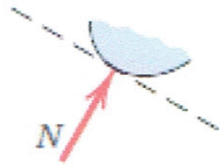
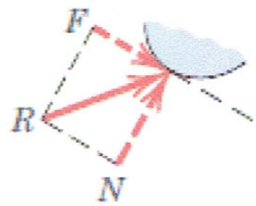
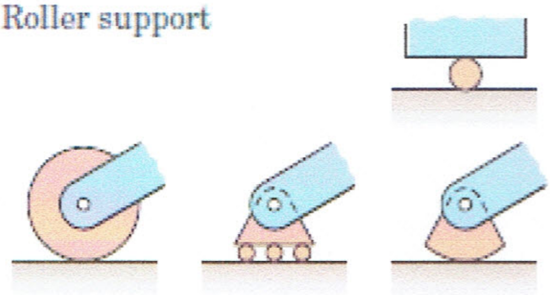
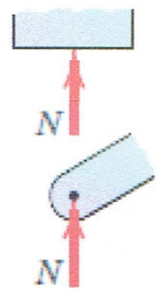
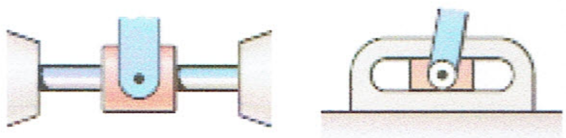
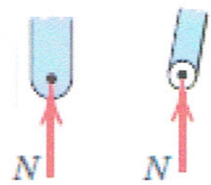
① قوة عمودية ② قوة افقية ③ عزم تدوير



④ Smooth surface — يعوض عنه بقوة واحدة عمودية على سطح الاسناد



MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)	
Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action   	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

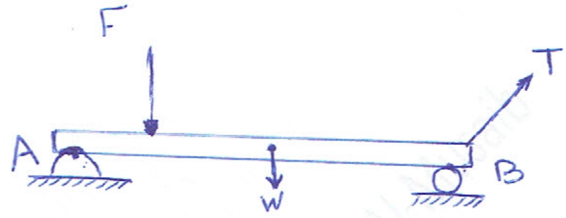
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

مخطط الجسم الحر

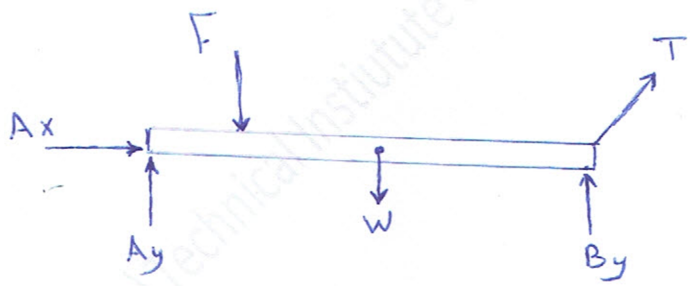
* Free Body Diagram

* Draw the F.B.D for the following problem.

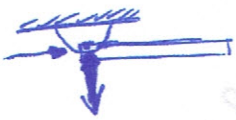
①



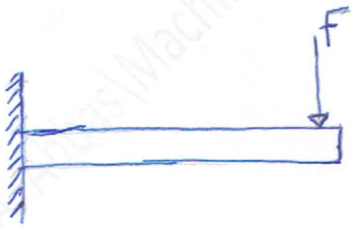
Solution :-

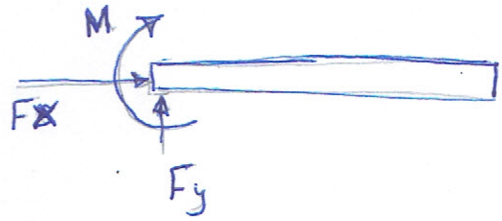


F.B.D



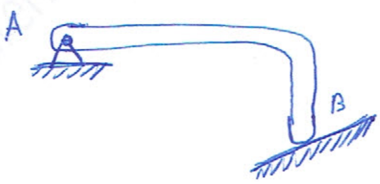
②

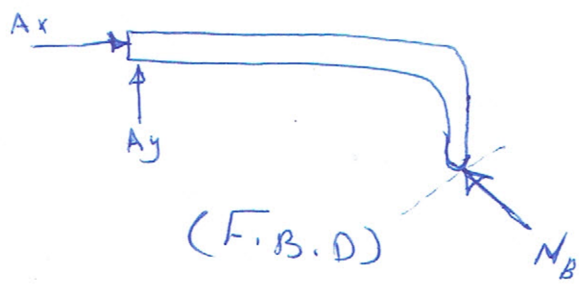




(F.B.D)

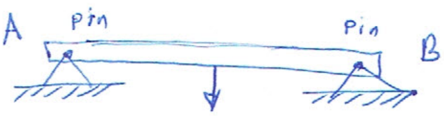
③

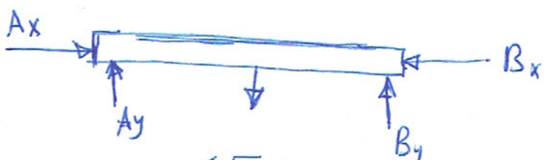




(F.B.D)

④





(F.B.D)

Note: When a body is in equilibrium, the resultant of all forces acting on it «zero»

$\sum F_x = 0$ ---- ①

$\sum F_y = 0$ --- ②

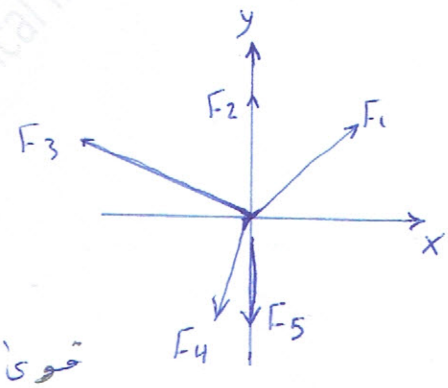
$\sum M_o = 0$ --- ③

---

① Equilibrium of Concurrent Forces

$\sum F_x = 0$

$\sum F_y = 0$



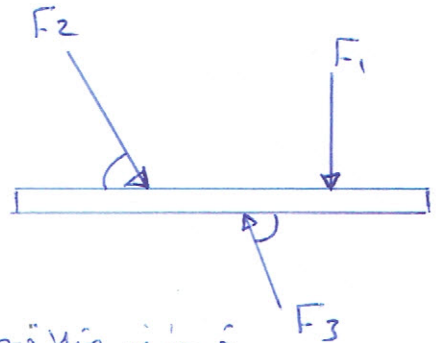
قوى متلاقية
«Concurrent Forces»

② Equilibrium of non-Concurrent Forces

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M_o = 0$



قوى غير متلاقية
«Non-Concurrent Forces»

* The equilibrium of concurrent and coplanar Forces:

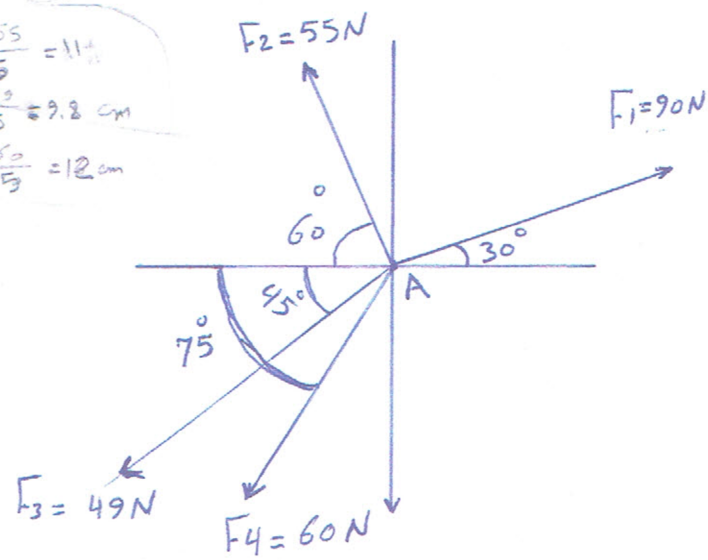
① Using Graphical method

* of the Forces concurrent, coplanar Can say the body is equilibrium when the rectangular Forces is Closed

* If the tow Forces acting on the body and equal magnitude (بمقدار متساوي) with opposite direction (واتجاه متعاكس) and same line action (وبنفس خط الفعل), the rectangular Forces is as the line. (استطالة القوى سوف تكون خط مستقيم)

F2 — A — F1

EX1:- Are the Forces acting at point A shown in the Figure below in equilibrium or not? (use The Graphical method)



Sol:-

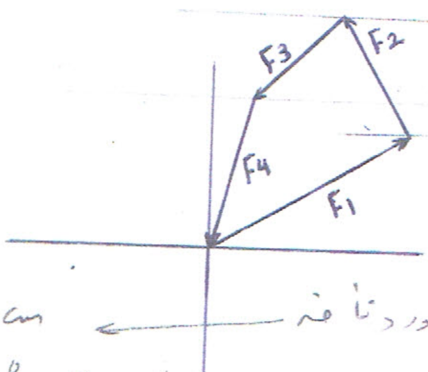
(على البورد) 5 N = 1 cm, $F_1 = \frac{90}{5} = 18$ cm, $F_2 = \frac{55}{5} = 11$, $F_3 = \frac{49}{5} = 9.8$ cm, $F_4 = \frac{60}{5} = 12$ cm

① scale : 30 N = 1 cm

$F_1 = \frac{90}{30} = 3$ cm, $F_2 = 1.83$ cm

$F_3 = 1.63$ cm, $F_4 = 2$ cm

② Draw the rectangular forces

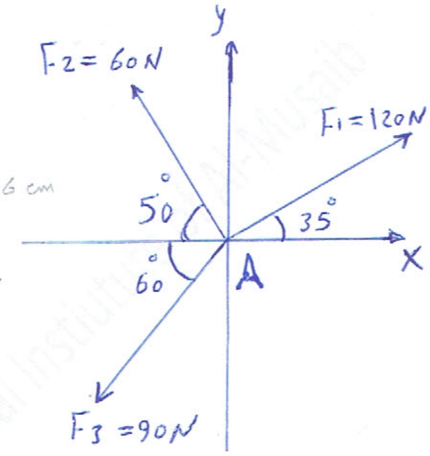


From the Draw besid notes the rectangular Forces is closed, so can say it equilibrium state.

على البورد نأخذ 5 N = 1 cm

$F_1 = \frac{90}{5} = 18$ cm, $F_2 = 11$ cm, $F_3 = 9.8$, $F_4 = 12$ cm

EX 2:- Are the Forces F_1, F_2, F_3 which acting at the point « A » in equilibruim or not, in the Figure below. using graphical method.



Solu:

① Take scale for the Draw

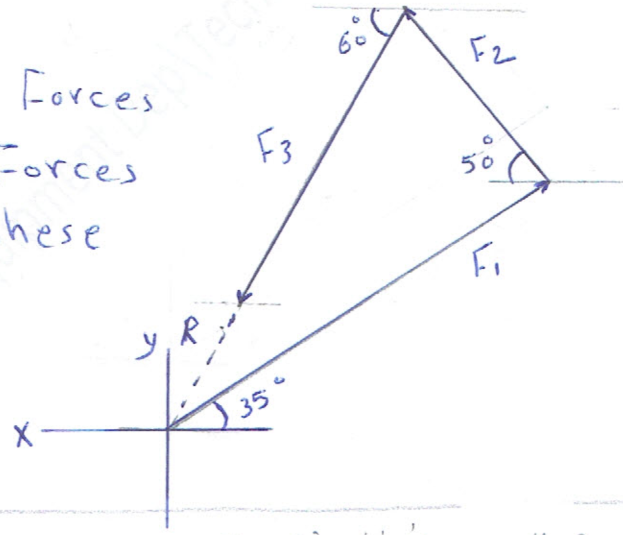
20 N = 1 cm

10N = 1 cm $F_1 = \frac{120}{10} = 12$ $F_2 = \frac{60}{10} = 6$ cm
على البورد $F_3 = \frac{90}{10} = 9$ cm

$\therefore F_1 = \frac{120}{20} = 6$ cm

$F_2 = 3$ cm $F_3 = 4.5$ cm

② Draw the rectangular Forces
can see the rectangular Forces not closed, that mean these Forces not in equilibruim.



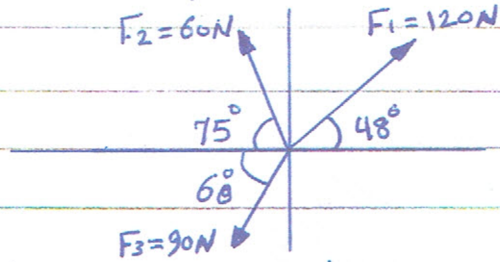
على البورد نأخذ مقياس رسم

5N = 1 cm

$F_1 = \frac{120}{5} = 24$ cm $F_2 = \frac{60}{5} = 12$ cm , $F_3 = \frac{90}{5} = 18$ cm

② Using Analytical Method

Example 3: Show that the three Forces F_1, F_2 and F_3 shown in Fig below are in equilibrium.



Solution:

$R_x = \sum F_x = 0$ ——— ①

$R_y = \sum F_y = 0$ ——— ②

$\sum F_x = 0$

$F_1 \cos\theta_1 - F_2 \cos\theta_2 - F_3 \cos\theta_3 = 0$

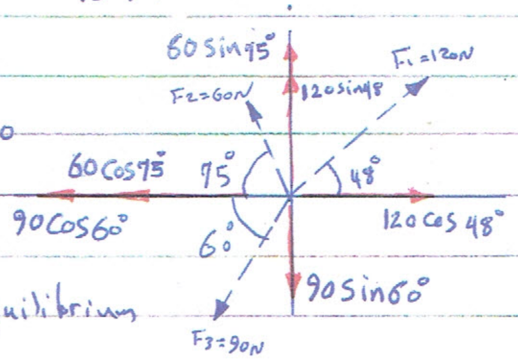
$120 \cos 48° - 60 \cos 75° - 90 \cos 60° = 0$

$80.3 - 15.6 - 45 = 19.7 \neq 0$

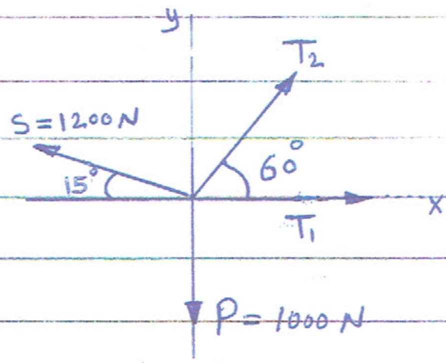
$\therefore$ since $\sum F_x = 19.7$ and $19.7 \neq 0$

so the Forces F_1, F_2 and F_3 are not in equilibrium

no need to continue



EX 4: Find the magnitude of the tension Forces T_1 and T_2 which makes the Fig below in equilibrium



<u>Solution:</u>

$\sum F_x = 0$

$T_1 + T_2 \cos 60° - 1200 \cos 15° = 0$

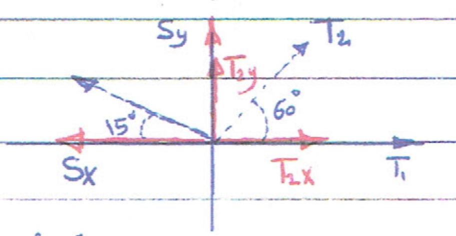
$T_1 + 0.5 T_2 - 1160 = 0$

$\therefore \boxed{T_1 = 1160 - 0.5 T_2}$ ——— ①

$\sum F_y = 0$

$T_2 \sin 60° + 1200 \sin 15° - 1000 = 0$

$0.866 T_2 + 310.5 - 1000 = 0$



$\therefore T_2 = \dfrac{689}{0.866} = \boxed{796\ N}$ نعوض بمعادلة ① لإيجاد T_1

$\therefore T_1 = 1160 - (0.5 * 796) = \boxed{762\ N}$

Other solution of Ex4 :-

① $\sum F_y = 0$

$-P + T_{2y} + S_y = 0$

$-1000 + T_2 \sin 60^\circ + 1200 \sin 15^\circ = 0$

$-1000 + 0.866 T_2 + 310.6 = 0$

$1000 - 310.6 = 0.866 T_2$

$\therefore T_2 = \frac{689.4}{0.866} = \boxed{796 \ N}$

② $\sum F_x = 0$

$T_1 + T_2 \cos 60 - 1200 \cos 15 = 0$

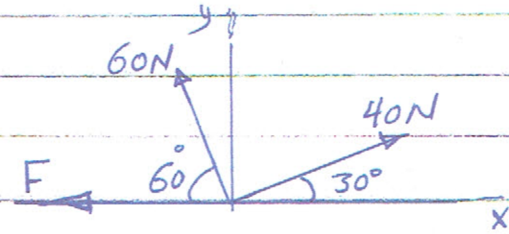
$T_1 + 0.5 T_2 - 1160 = 0$

$T_1 + (0.5 * 796) - 1160 = 0$

$\therefore T_1 = -398 + 1160 = \boxed{762 \ N}$

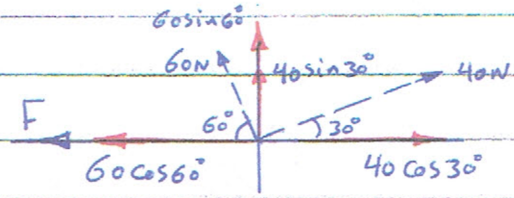
Example 5: Determine the Force (F) which make the Forces system in equilibrium.

Solution:



$\Sigma F_x = 0$

$-F + 40\cos 30° - 60\sin 60° = 0$

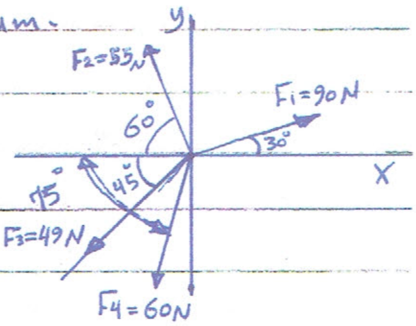


$-F + 34.64 - 30 = 0$

$\therefore F = 34.64 - 30 = 4.64\ N$

Example 6: show analytically if the system of forces in the Fig, below is equilibrium.

Solution:



$R_x = \Sigma F_x = 0$

$\underset{F_{1x}}{90\cos 30°} - \underset{F_{2x}}{55\cos 60°} - \underset{F_{3x}}{49\cos 45°} - \underset{F_{4x}}{60\cos 75°} = 0$

$77.9 - 27.5 - 34.7 - 15.6 = 0.1 \simeq 0$

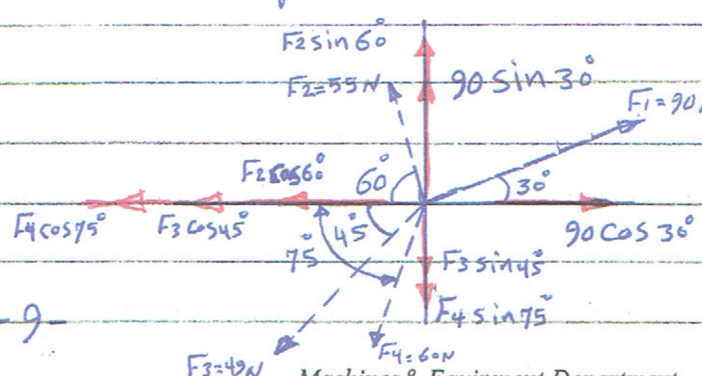
$R_y = \Sigma F_y = 0$

$\underset{F_{1y}}{90\sin 30} + \underset{F_{2y}}{55\sin 60°} - \underset{F_{3y}}{49\sin 45} - \underset{F_{4y}}{60\sin 75°} = 0$

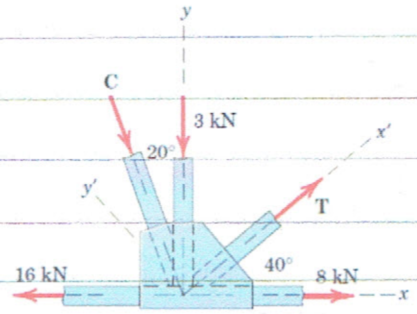
$45 + 47.7 - 34.7 - 58 = 0$

since both $R_x = 0$ and $R_y = 0$

$\therefore$ the system of forces is in equilibrium.



Example 7: The Forces on the gusset plate of a joint in a bridge truss act as shown in figure below. Determine the forces "C" and "T" to maintain equilibrium of the joint.



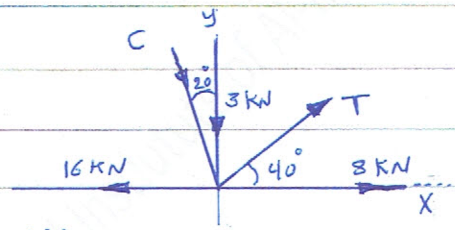
Solution:

$\sum F_x = 0$

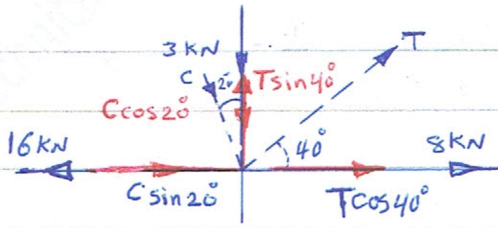
$8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$

$8 + 0.766T + 0.35C - 16 = 0$

$0.766T + 0.35C - 8 = 0$



$0.766T + 0.35C = 8$ ——— (1)



$\sum F_y = 0$

$T\sin 40^\circ - C\cos 20^\circ - 3 = 0$

$0.65T - 0.94C = 3$ ——— (2)

From Equ (1), (2) we get   طريقة الحل بالتعويض

$T = 9.05\ KN$, $C = 3.06\ KN$

* Note that: From Equ (1)

$0.766T + 0.35C = 8$ قيمة T تعوض بمعادلة (2) الاعلاه

$0.766T = 8 - 0.35C \Rightarrow T = \dfrac{8 - 0.35C}{0.766} = 10.45 - 0.46C$ ——— (3)

$0.65(10.45 - 0.46C) - 0.94C = 3$

$6.793 - 0.299 - 0.94C = 3$

قيمة C تعوض بمعادلة (1) او (2) الاعلاه

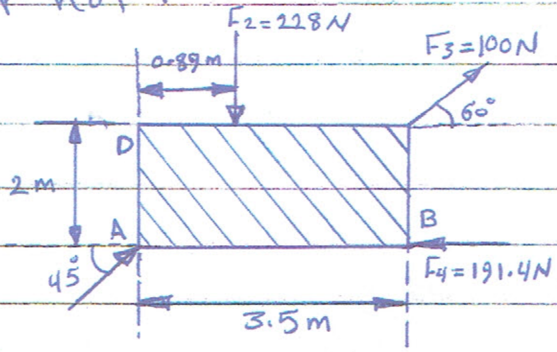
$6.793 - 1.239C = 3 \Rightarrow C = \dfrac{6.793 - 3}{1.239} = 3.06\ KN$

$0.766T + 0.35C = 8$

$0.766T + 0.35(3.06) = 8$

$0.766T + 1.071 = 8 \Rightarrow T = \dfrac{8 - 1.071}{0.766} = 9.05\ KN$

Example 8: Find if the Figure below which acting to it many Forces in equilibrium or not.



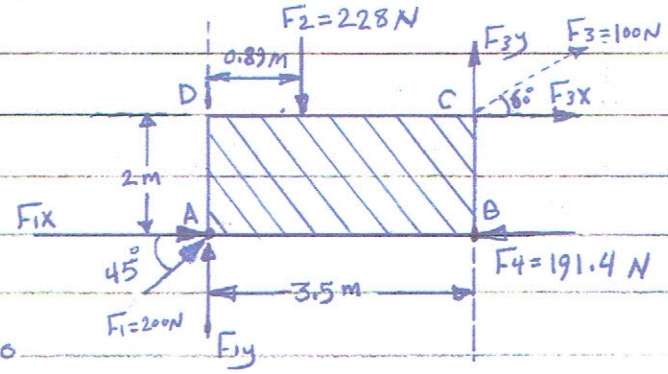
<u>solution:</u>

① $R_x = \Sigma F_x = 0$

$F_{1x} - F_4 + F_{3x} = 0$

$F_1 \cos 45° - 191.4 + F_3 \cos 60° = 0$

$141.4 - 191.4 + 50 = 0$



« F.B.D »

② $R_y = \Sigma F_y = 0$

$F_{1y} + F_{3y} - F_2 = 0$

$F_1 \sin 45° + F_3 \sin 60° - 228 = 0$

$141.4 + 86.6 - 228 = 0$

③ $\Sigma M_A = 0$

$\underset{F*d}{F_2 * 0.89} - \underset{F*d}{F_{3y} * 3.5} + \underset{F*d}{F_{3X} * 2} = 0$

$228 * 0.89 - 86.6 * 3.5 + 50 * 2 = 0$

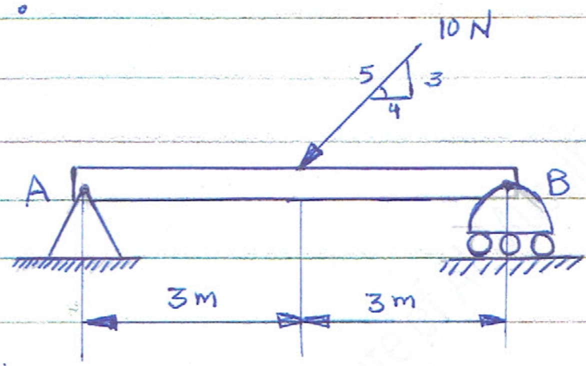
$203 - 303 + 100 = 0$

Since $R_x = 0$, $R_y = 0$ and $\Sigma M_A = 0$

so the system in equilibrium

* Equilibrium of non – Concurrent forces

Example 9: From the beam shown below, Find all the reactions.



Solution:

① $\sum M_A = 0$

F * d F * d

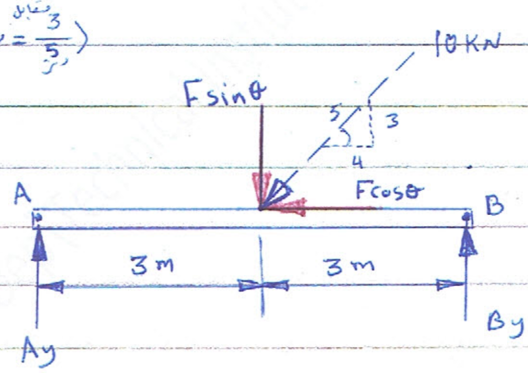
$(F\sin\theta * 3) - (B_y * 6) = 0$

($\sin\theta = \frac{3}{5}$) مقابل / وتر

$(10 * \frac{3}{5}) * 3 - 6B_y = 0$

$18 = 6B_y$

$\therefore B_y = \frac{18}{6} = 3\,N \uparrow +$



(F.B.D)

② $\sum F_y = 0$

$B_y - F\sin\theta + A_y = 0$

$3 - (10 * \frac{3}{5}) + A_y = 0$

$3 - 6 + A_y = 0$

$\therefore A_y = 3\,N \uparrow +$

③ $\sum F_x = 0$

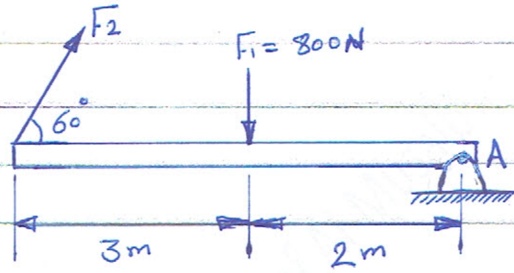
$A_x - (F\cos\theta) = 0$

($\cos\theta = \frac{4}{5}$) مجاور / وتر

$A_x - (10 * \frac{4}{5}) = 0$

$\therefore A_x = 8\,N \rightarrow +$

Example 10 :- For the Figure below, find the reaction at the support "A" and the magnitude of "F_2" which make the body in equilibrium.

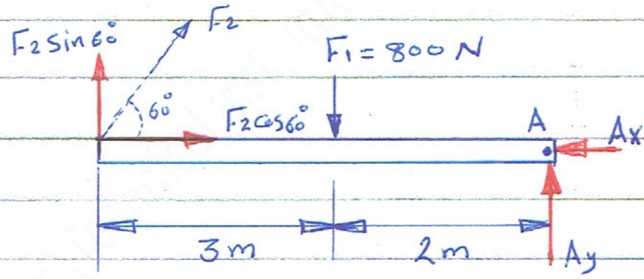


solution:

$\Sigma M_A = 0$

$(F_2 \sin 60) * 5 - 800 * 2 = 0$

$\therefore F_2 = \dfrac{800 * 2}{\sin 60 * 5} = \boxed{369.6\ N}$



« F.B.D »

$\Sigma F_y = 0$

$F_2 \sin 60 + A_y - 800 = 0$

$369.6 \sin 60 + A_y - 800 = 0$

$\therefore A_y = 800 - 369.6 \sin 60$

$= \boxed{480\ N}$

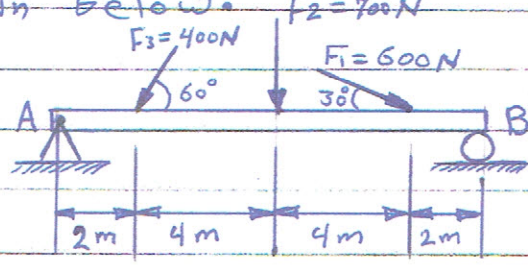
$\Sigma F_x = 0$

$F_2 \cos \theta - A_x = 0$

$\therefore A_x = F_2 \cos \theta$

$= 369.6 \cos 60° = \boxed{184.8\ N}$

Example 11: Find the reactions of the supports "A" and "B" of the body shown below.



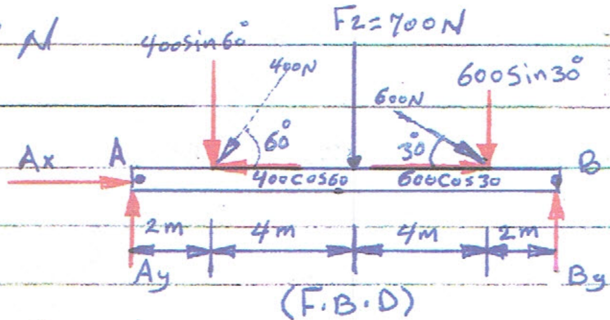
solution:

$\Sigma M_A = 0$

$(400 \sin 60) * 2 + 700 * 6 + (600 \sin 30) * 10 - B_y * 12 = 0$

$692.83 + 4200 + 3000 - 12 B_y = 0$

$\therefore B_y = \frac{7892.83}{12} = 657.8\ N$



$\Sigma M_B = 0$

$A_y * 12 - (400 \sin 60) * 10 - 700 * 6 - (600 \sin 30) * 2 = 0$

$12 A_y - 3464 - 4200 - 600 = 0$

$12 A_y - 8264 = 0$

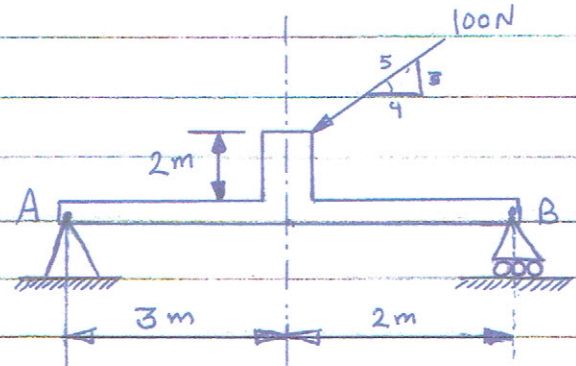
$\therefore A_y = \frac{8264}{12} = 688.7\ N$

$\Sigma F_x = 0$

$A_x - 400 \cos 60 + 600 \cos 30 = 0$

$\therefore A_x = -319.7\ N$

Example 12: From the figure below, compute all forces act on the beam « A B »



Solution:

$(\sin\theta = \frac{3}{5})$, $(\cos\theta = \frac{4}{5})$

① $\sum M_A = 0$

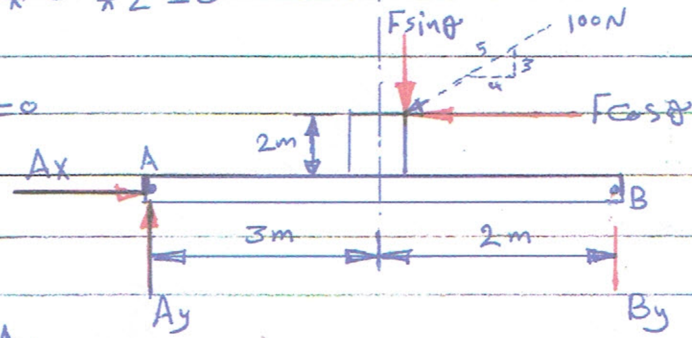
$(-B_y * 5) + (F\sin\theta * 3) - F\cos\theta * 2 = 0$

$(-5B_y) + (100 * \frac{3}{5} * 3) - (100 * \frac{4}{5} * 2) = 0$

$-5B_y + (60 * 3) - (80 * 2) = 0$

$-5B_y + 180 - 160 = 0$

$\therefore B_y = \frac{180-160}{5} = 4\,N \uparrow +$



« F.B.D »

② $\sum F_y = 0$

$A_y + B_y - 60 = 0$

$A_y + 4 - 60 = 0$

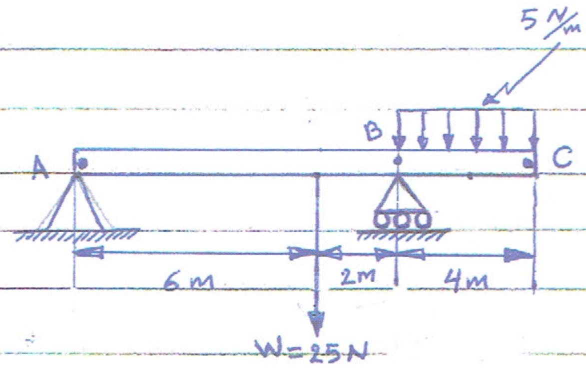
$\therefore A_y = 56\,N \uparrow +$

③ $\sum F_x = 0$

$A_x - 80 = 0$

$\therefore A_x = 80\,N \xrightarrow{+}$

Example 13: If the beam ABC weight (25 N), Compute the reactions at "A" and "B".



Solution:

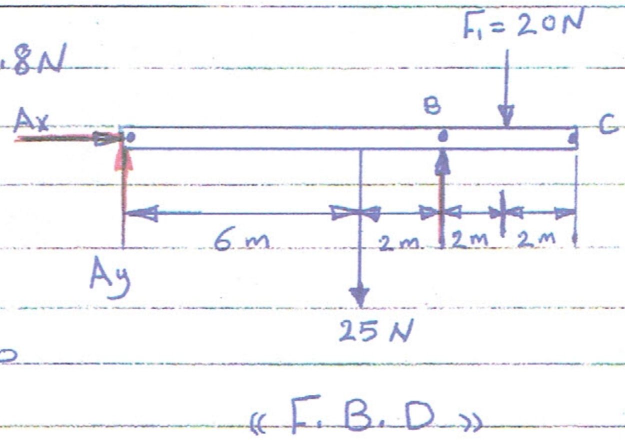
$F_1 = 5\ N/m * 4 = 20N$

① $\sum M_A = 0$

$20*10 - By*8 + 25*6 = 0$

$200 - 8By + 150 = 0$

$\therefore By = \frac{200+150}{8} = 43.8N$



② $\sum Fy = 0$

$Ay - 25 + By - 20 = 0$

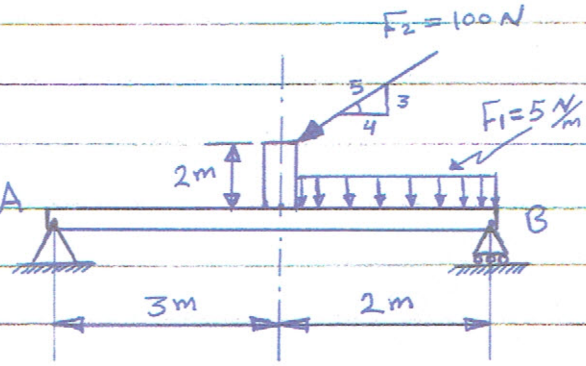
$Ay - 25 + 43.8 - 20 = 0$

$\therefore Ay = 1.25\ N \uparrow +$

③ $\sum Fx = 0$

$Ax = 0$

Example 14: From the figure below, Compute all the Forces act on the beam AB.



solution:

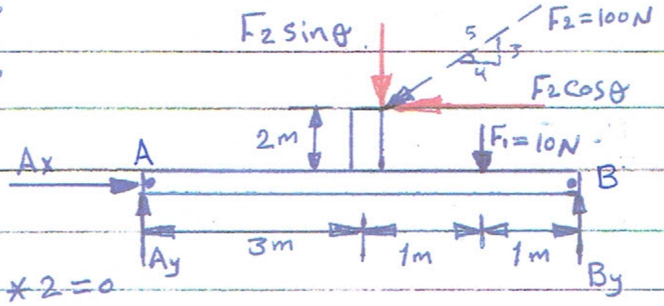
$F_1 = 5 \frac{N}{m} * 2m = 10 N$

$F_{2x} = F_2 \cos\theta \qquad (\cos\theta = \frac{4}{5})$

$= 100 * \frac{4}{5} = 80 N$

$F_{2y} = F_2 \sin\theta \qquad (\sin\theta = \frac{3}{5})$

$= 100 \times \frac{3}{5} = 60 N$



① $\Sigma M_A = 0$

$-B_y * 5 + 10 * 4 + 60 * 3 - 80 * 2 = 0$

$-5B_y + 40 + 180 - 160 = 0$

$-5B_y + 60 = 0$

$\therefore B_y = \frac{-60}{-5} = 12 N \uparrow +$

② $\Sigma F_y = 0$

$A_y + B_y - 60 - 10 = 0$

$A_y + 12 - 60 - 10 = 0$

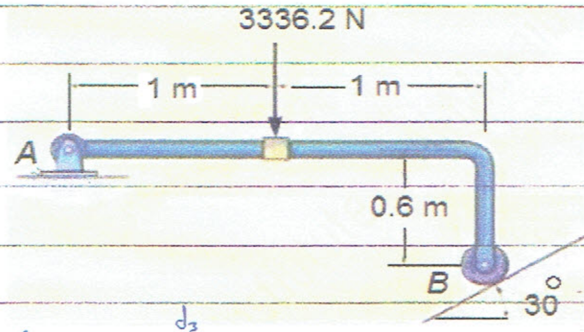
$\therefore A_y = 58 N \uparrow +$

③ $\Sigma F_x = 0$

$A_x - 80 = 0$

$\therefore A_x = 80 N \overset{+}{\rightarrow}$

Example 15 :- Determine the horizontal and vertical components of reaction on the member at the pin "A" and the normal reaction at the roller "B" in the Figure below.



solution:

① Draw the (F.B.D) of the Fig.

② $\sum M_A = 0$

N_{By} d_1 N_{BX} d_2 d_3

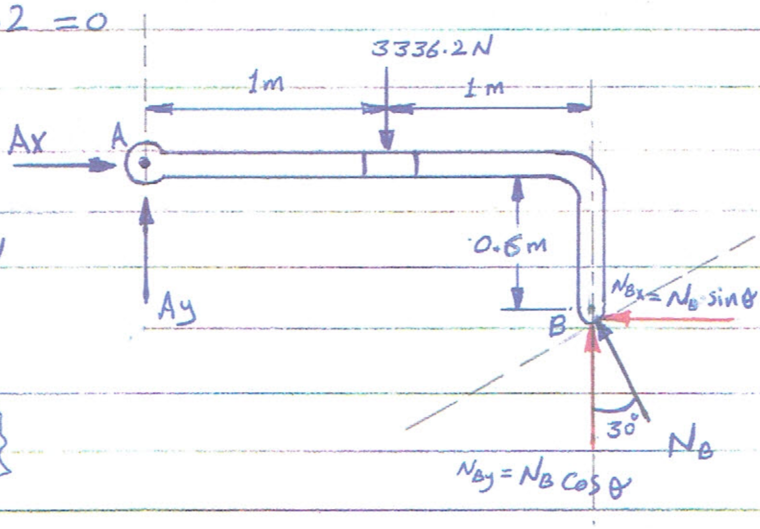
$(-N_B \cos 30°) * 2 + (N_B \sin 30°) * 0.6 + (3336.2 * 1) = 0$

$-1.74\, N_B + 0.3\, N_B + 3336.2 = 0$

$-1.44\, N_B = -3336.2$

$\therefore N_B = \dfrac{-3336.2}{-1.44} = 2316.8\ N$

$N_{BX} = N_B \sin 30° = 1158.4\ N$
$N_{By} = N_B \cos 30° = 2006.4\ N$



« F.B.D »

③ $\sum F_x = 0$

$A_x - (2316.8 \sin 30°) = 0$

$\therefore A_x = 1158.4\ N$

④ $\sum F_y = 0$

$A_y + (2316.8 \cos 30°) - 3336.2 = 0$

$A_y + 2006.4 - 3336.2 = 0$

$\therefore A_y = 1329.8\ N$